allow a representation by Equations (4) and (5).

For polymers we replace $1/p$ by the molal weight, and so instead of (2) we have

$$t = (1/r)\log(M/M_m) \qquad (9)$$

Similar substitutions are made in (6), (7), and (8). In addition, we prefer to use weight fractions rather than mole fractions so that X in Equation (5) is replaced by the cumulative weight fraction W and x_i in (6) to (8) by the weight fraction w_i. It makes little difference which kind of variable we choose; indeed, it can be shown that a log-normal distribution for weight fractions implies a log-normal distribution for mole fractions with the same standard deviation (6).

Various average molal weights are of interest for polymers. In order to obtain a general result, we define the average molal weight of order g by

$$(M_g)^g = \int_{-\infty}^{\infty} M^g f d\log M \qquad (10)$$

We introduce the nearly normal frequency function (4) (after replacing $1/p$ by M) into (10) and obtain after a slightly involved calculation* for the average molal weight of order g:

$$\log M_g = \log M_m + 1.152\, g r^2 + g^{-1}\log[1 - h_3(2.3\, g r)^3 + h_4(2.3\, g r)^4] \qquad (11)$$

Thus we have quite convenient expressions for the number average molal weight $M_n (g = -1)$:

$$\log M_n = \log M_m - 1.152\, r^2 - \log[1 + h_3(2.3r)^3 + h_4(2.3r)^4], \qquad (12)$$

for the viscosity molal weight by writing the exponent a of the relation of Mark and Houwink for g in (11), and for the weight average molal weight $M_w (g = 1)$

$$\log M_w = \log M_m + 1.152\, r^2 + \log[1 - h_3(2.3r)^3 + h_4(2.3r)^4] \qquad (13)$$

The relations given by Chiang (5) for the log-normal distribution follow from (11) with $h_3 = h_4 = 0$.

In general one will interpret the terms containing h_3 and h_4 in all these relations as correction terms, representing small deviations from the log-normal distribution. In other words, h_3 and h_4 should be small compared with unity. Practically the permissible limits of h_3 and h_4 will depend on the magnitude of r and also on the specific application.

AVERAGE VAPOR PRESSURES

Returning to oil mixtures, we define the average ideal vapor pressure and the average reciprocal ideal vapor pressure by

$$p_a = \sum_i x_i p_i; \quad 1/p_r = \sum_i x_i/p_i \qquad (14)$$

The reciprocal vapor pressure determines the dew point of the mixture. Replacing M by $1/p$, we obtain from (13) and (12)

$$\log p_a = \log p_m + 1.151 r^2 + \log[1 + h_3(2.3r)^3 + h_4(2.3r)^4] \qquad (15)$$

$$\log p_r = \log p_m - 1.151 r^2 - \log[1 - h_3(2.3r)^3 + h_4(2.3r)^4] \qquad (16)$$

NOTATION

G = Gaussian [Equation (1)]
M = molal weight
M_g = average molal weight of order g
M_m = geometric average molal weight
M_n = number average molal weight
M_w = weight average molal weight
W = cumulative weight fraction
X = cumulative mole fraction
f = frequency function [Equation (4)]
h_3, h_4 = coefficients of the Hermitian terms
p = vapor pressure
p_i = vapor pressure of species i
p_m = geometric mean vapor pressure
p_a = average vapor pressure
p_r = average reciprocal vapor pressure
r = standard deviation
t = reduced distribution variable [Equations (2), (9)]
w_i = weight fraction of species i
x_i = mole fraction of species i

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[*] We substitute t for M with the aid of (9), transform to a new variable $y = t - 2.303\, gr$, and order the polynomial into Hermitian polynomials. Then the integration is easy because of the relation between Hermitian polynomials and the derivatives of G.

Liquid-side Mass Transfer Coefficients in Packed Towers

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In the June, 1959, issue of the *A.I.Ch.E. Journal*, Onda, Sada, and Murase (4) presented a correlation for liquid-phase mass transfer coefficients based on values of k_L for Raschig rings obtained by dividing the volumetric coefficient, by the wetted area of Fujita (2). Since the authors showed that their correlation does not provide insight into the mechanism of absorption, because the same correlation is obtained with either the two-film theory or the penetration theory assumed, the only justification for the correlation is that it is more convenient or can be used where other design methods fail.

For design purposes the correlation must be used with the wetted-area data of Fujita (2) which limits its application to Raschig rings and water systems. In a recent communication Professor Onda (5) mentions having obtained an equation for the wetted area of Berl saddles which was used to check his correlation for Raschig rings. He found that he could use the same type of correlation, but the constant for Berl saddles was 0.018 instead of the 0.013 recommended for Raschig rings. It is obvious then that the correlation is not of a general enough nature to be useful for extension to other packing systems. For design purposes the equations of Sherwood and Holloway (6) are more convenient and thoroughly tested.

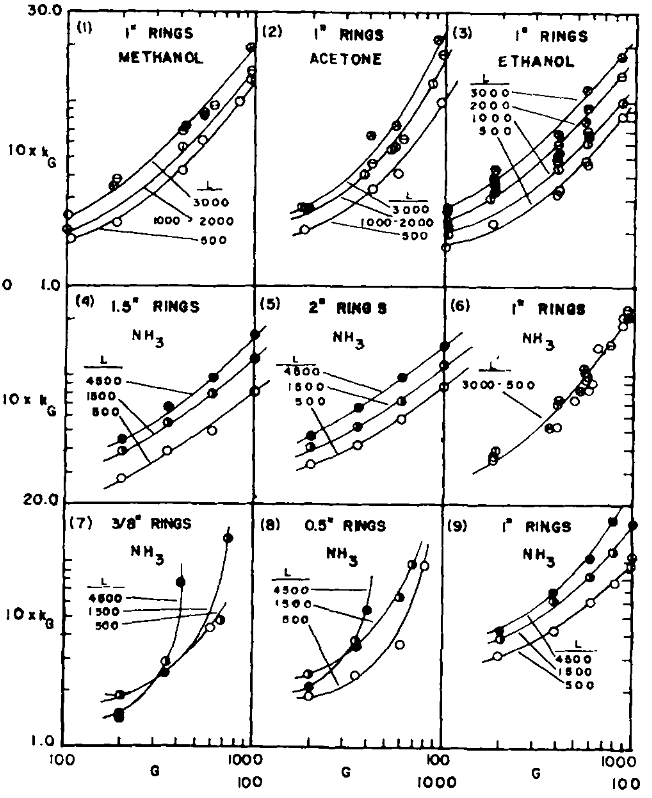

Fig. 1. k_G obtained from k_Ga for Raschig ring, air-water-solute systems: 1, 2, 3, and 6 from Houston and Walker (3), and 4, 5, 7, 8, and 9 from Fellinger (1).

The authors claim that their correlation supports the assumption that the effective interfacial area is directly proportional to the wetted areas which they employed. If this were true, it should be possible to use a_w to extract k_G from k_Ga and obtain a correlation in terms of the gas-phase Reynolds and Schmidt numbers and/or other dimensionless groups. To test this possibility the Raschig-ring data of Fellinger (1) (corrected for liquid-phase resistance) and Houston and Walker (3) were divided by the appropriate Fujita value of a_w to extract k_G, which is plotted in Figure 1 for nine systems. An examination of the figure indicates that any correlation of the k_G values will show k_G as a function of liquid rate. If a_w was directly proportional to the true a, the resulting values of k_G would be almost independent of the liquid rate. In addition the curvature of the lines implies that no simple function of the gas rate would be suitable for a correlation. On the basis of these two observations it is possible to conclude that wetted areas are not equal to, or directly proportional to, the true effective areas in packed absorption towers.

A basic weakness in the authors' approach to the problem is the assumption that the effective interfacial area is independent of gas rate. This assumption is based on extensive k_La data taken at very low gas rates with the exception of a few points by Sherwood and Holloway (6) for 1.5-in. rings. Shulman and co-workers (7) pointed out how an erroneous conclusion can be reached because the few points taken at high gas rates happen to be in a region where a is just about independent of gas rate. Moreover their data for a (7) indicate a strong dependence on gas rate for the smaller packings, such as the rings used in the paper under discussion.

Correlations for k_L and k_G based on the same effective interfacial areas were published by Shulman and coworkers (7) some time ago. Recently Yoshida and Koyanagi (8) determined effective interfacial areas, by means of an entirely different method, which were in agreement. In both cases a seemed to be unrelated to a_w. Literature surveys (8) show greater disagreement in the extensive data for a_w than in the data for a.

Enough basic work on mass transfer rates has been done to predict that major advances in the understanding of packed columns must come from studies of effective interfacial areas, rather than from the reworking of data obtained from columns which are not representative of those in use.

NOTATION

a = effective interfacial area, sq.ft./cu.ft.
a_w = wetted area, sq.ft./cu.ft.
G = superficial gas rate, lb./(hr.)(sq.ft.)
k_G = gas-phase mass transfer coefficient, lb. moles/(hr.)(sq.ft.)(atm.)
k_L = liquid-phase mass transfer coefficient, lb. moles/(hr.)(sq.ft.)(lb.moles/cu.ft.)
L = superficial liquid rate, lb./(hr.)(sq.ft.)

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